Chinese Journal of Aeronautics, (2016), **29**(1): 248–256

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

A new method of single celestial-body sun positioning based on theory of mechanisms

Zhang Wei [a,b,*], **Xu Xiaofeng** [c], **Wu Yuanzhe** [d]

[a] *Airport College, Civil Aviation University of China, Tianjin 300300, China*
[b] *Aviation Ground Special Equipment Research Base, Tianjin 300300, China*
[c] *Aeronautical Automation College, Civil Aviation University of China, Tianjin 300300, China*
[d] *Aeronautical Engineering College, Civil Aviation University of China, Tianjin 300300, China*

Received 23 April 2015; revised 18 September 2015; accepted 22 October 2015
Available online 22 December 2015

KEYWORDS

Celestial navigation;
Inverse kinematics;
Longitude and latitude coordinates;
Mechanisms;
Navigation systems;
Position measurement;
Single celestial-body positioning;
Sun

Abstract Considering defects of current single celestial-body positioning methods such as discontinuity and long period, a new sun positioning algorithm is herein put forward. Instead of traditional astronomical spherical trigonometry and celestial coordinate system, the proposed new positioning algorithm is built by theory of mechanisms. Based on previously derived solar vector equations (from a $C^1R^2P^2$ series mechanism), a further global positioning method is developed by inverse kinematics. The longitude and latitude coordinates expressed by Greenwich mean time (GMT) and solar vector in local coordinate system are formulated. Meanwhile, elimination method of multiple solutions, errors of longitude and latitude calculation are given. In addition, this algorithm has been integrated successfully into a mobile phone application to visualize sun positioning process. Results of theoretical verification and smart phone's test demonstrate the validity of presented coordinate's expressions. Precision is shown as equivalent to current works and is acceptable to civil aviation requirement. This new method solves long-period problem in sun sight running fixing and improves applicability of sun positioning. Its methodology can inspire development of new sun positioning device. It would be more applicable to be combined with inertial navigation systems for overcoming discontinuity of celestial navigation systems and accumulative errors of inertial navigation systems.

1. Introduction

Despite a widespread application of global navigation satellite systems (GNSS), celestial positioning has still held an important position no matter as an independent navigation method or as a part of an integrated navigation system.[1–4] It is well accepted that celestial navigation is capable of full autonomy

* Corresponding author. Tel.: +86 22 24092477.
E-mail address: drwadecheung@gmail.com (W. Zhang).
Peer review under responsibility of Editorial Committee of CJA.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2015.12.009

and strong anti-interference. To both military aviation and civil aviation, autonomy and anti-interference are important to a navigation system. According to recommendation of next-generation navigation, performance based navigation (PBN) has been proposed by the International Civil Aviation Organization (ICAO). PBN demands an autonomous navigation, including: the warning precision, which is defined as integrity monitoring alert limit (IMAL), is 0.3 n miles at approach phase, 2.0 n miles in air route and 1.0 n miles in terminal area.[5] With this precision, celestial navigation system is a suitable solution or a backup at least.

Among many celestial navigation methods, sun positioning is known as the oldest single celestial-body navigation method. It is also regarded as an important foundation of other methods. As a kind of celestial navigation technology, sun positioning is a reliable way to obtain latitudes and longitudes of observation points. And, it is only based on single star's position and orientation information, such as Polaris and solar vectors. It is mainly used to calculate the longitude and latitude coordinates of observation place by the sun direction in earth surface coordinate, including azimuth angle and elevation angle.[6] In recent years, primary research results of single celestial positioning are as follows. Wang et al.[7] reached the conclusion that the position lines of azimuth difference and height difference can be used for the single celestial positioning. Tang and Yang[8] obtained their position information with a single celestial altitude and azimuth information, which overcomes defects of long period and low precision in sun sight running fixing. Jiang and Yin[9] put forward a new method to render the dynamic scene to simulate celestial fixing principle of sextant in navigation and presented a mathematical model of celestial fixing by sextant. It improved that single celestial navigation is an automatic process. Jia et al.[10] realized sun sight running fixing with computer-aided calculating transcendental equation and accuracy compensation theory. Its method can save time and improve the positioning betimes greatly. Based on sun's azimuth angle and elevation angle, Yuan and Zhang[11] designed a positioning system to achieve a semi-automatic single celestial positioning. Wang et al.[12] analyzed relationship between polarization distribution of skylight and sun orientations and put forward a new mode of celestial navigation based on measuring skylight polarization at two mutually perpendicular orientations. Peng et al.[13] built an automatic calculation system of navigation astronomical orientation by C sharp tool and altitude difference principle, which improved speed and precision of calculation.

It can be seen that many studies focused on improving detection instruments' precision and exploring new technologies, such as the high-precision charge-coupled device (CCD) star sensors and the integrated navigation systems.[2,3] Few studies centered on principle and algorithm of single celestial positioning. Besides, there are problems in current single celestial positioning algorithms. For one thing, we always derive and calculate position information depending on traditional celestial coordinates, which need apparent motion of planets, spherical triangle, and some experience-based error corrections.[6,8] For another, a certain principle error happens because of curves replaced by straight lines in process of height difference calculation.[14] Moreover, if we consider formula errors in single celestial navigation process, positioning error in general moments is far more than the error in transit moment. The "transit moment limitation" in traditional calculation results in discontinuous navigation information,[6,15] so the real time positioning can be implemented in this way.

To solve problems among the existing single celestial positioning method, calculation framework of solar positioning is reconstructed by theory of mechanisms. Wang[16] and many researchers testified the dependence of solar vectors on coordinates and time of observation point. Authors have introduced a proven $C^1R^2P^2$ mechanism, whose forward kinematics can be used for calculation of solar vector. In an inverse way, latitude and longitude expressions of observation point in the Greenwich mean time (GMT) can be derived by inverse kinematics of mechanisms[17,18] with a given solar vector. In a word, if time information (year, month, date, hour, minute and second) and its corresponding solar vector in an observation point are known, latitude and longitude of this point are solvable.

Since every variable is related to an explicit joint motion defined by theory of mechanisms, the proposed calculation method is likely to be more comprehendible. Based on those achieved joint motions, a positioning solution is mathematically easy to get by solar vectors. A new method is raised to break through traditional celestial coordinate calculation with astronomical trigonometry, overcome the limitations of sun sight running fixing (obtaining observation's position by observing sun's position at two different moments), achieve continuous positioning at a long time, and improve practicability of single celestial-body positioning applications. In addition, this method utilizes only GMT instead of local time in calculation of latitude and longitude. So it can remove the timing error in radio time service process and improve the positioning precision. To reduce the error of the proposed method, it can be used to combine with inertial navigation equipment and random weighting estimation method.[19]

2. A sun positioning algorithm based on theory of mechanisms

A new calculation model of sun positioning can be constructed with a serial mechanism $C^1R^2P^2$, as shown in Fig. 1. We use five kinematic joints of $C^1R^2P^2$ mechanism to simulate the revolution and rotation of earth to the sun. The rotation joint θ_1 stands for the revolution of earth around sun in the ecliptic plane. The prismatic joint d_2 stands for radius change of earth revolution, corresponding to the elliptical orbit. The rotation joint θ_3 stands for reversing rotation corresponding to revolution angle, in order to maintain constant direction of the axis of earth ($\theta_3 = -\theta_1$). The cylindrical joint can be divided into a prismatic joint d_4 and a rotation joint θ_5, stands for a rotation sleeve along polar axis of the earth. The prismatic joint d_6 stands for radius change on latitude circle of the earth. The joint variables d_4, θ_5 and d_6 are used together to simulate an arbitrary site on earth's surface. d_7 has a very tiny value to permit free moving of joint d_4 along the polar axis. The coordinate $oxyz$ is the earth surface coordinate, in which, x stands for positive west, y positive south, and z zenith. According to authors' previous study, solar vector expressions $\boldsymbol{S}$ in $oxyz$ coordinate can be achieved by product of exponentials formula method[20] or Denavit-Hartenberg method[21] in the form of

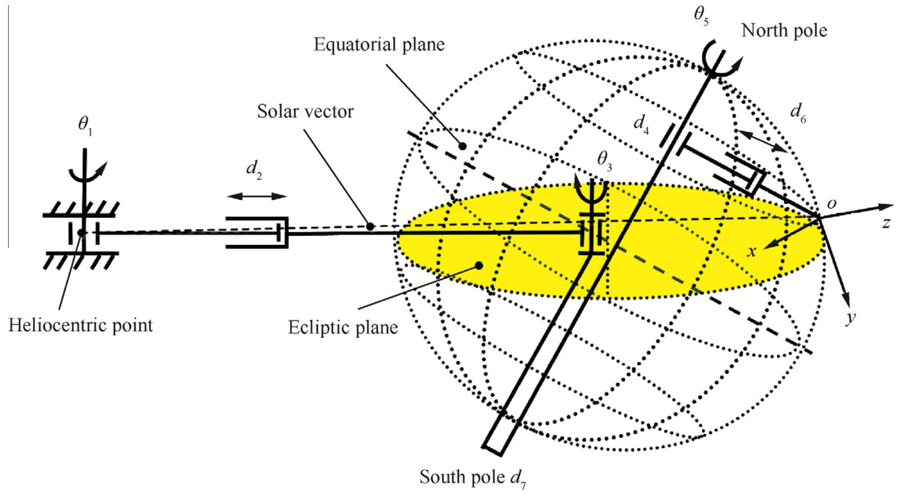

Fig. 1 A serial mechanism for sun positioning calculation.

$$\boldsymbol{S}=\begin{bmatrix}p_x\\p_y\\p_z\end{bmatrix}=\begin{bmatrix}d_2(\sin\theta_1\cos\theta_5-\cos\theta_1\sin\theta_5\cos\alpha)\\d_2\cos\theta_1\sin\alpha+d_4(\sin\theta_1\sin\theta_5+\cos\theta_1\cos\theta_5\cos\alpha)\\-d_2(\sin\theta_1\sin\theta_5-\cos\theta_1\cos\theta_5\cos\alpha)-d_6\end{bmatrix}\tag{1}$$

where p_x, p_y and p_z are the projections of solar vector in directions x, y and z respectively in the coordinate system $oxyz$. $\alpha = 23°26'$ is the angle between ecliptic plane and equatorial plane. θ_1 can be calculated by summing up the number of days (N) away from the date of perihelion and calculating the relationship expression between θ_1 and N. Because sun-earth distance d_2 is much greater than distance d_6, d_6 is then ignored. Further Eq. (1), one has

$$x=\sin\theta_1\cos\theta_5-\cos\theta_1\sin\theta_5\cos\alpha\tag{2}$$

$$y=\cos\theta_1\sin\alpha\cos\psi-\sin\psi(\sin\theta_1\sin\theta_5+\cos\theta_1\cos\theta_5\cos\alpha)\tag{3}$$

$$z=-\cos\theta_1\sin\alpha\sin\psi-\cos\psi(\sin\theta_1\sin\theta_5+\cos\theta_1\cos\theta_5\cos\alpha)\tag{4}$$

where the variable ψ represents the latitude.

2.1. Calculation of latitude

To achieve latitude from Eqs. (2)–(4), Euler transform solution and double variables arc tangent function[22] are applied to calculation. The detailed calculation process is as follows.

Firstly, define

$$a=\cos\theta_1\cos\alpha, b=\sin\theta_1$$

Hence, Eq. (2) becomes

$$x=b\cos\theta_5-a\sin\theta_5$$

Let $a=\rho\cos\varphi$, $b=\rho\sin\varphi$, where $\rho=\sqrt{a^2+b^2}$, $\varphi=\mathrm{atan2}(b,a)$, so we have

$$\sin(\varphi-\theta_5)=\frac{x}{\rho},\cos(\varphi-\theta_5)=\pm\sqrt{1-\frac{x^2}{\rho^2}}$$

At a further step,

$$\varphi-\theta_5=\mathrm{atan2}\left(\frac{x}{\rho},\pm\sqrt{1-\frac{x^2}{\rho^2}}\right)$$

Then,

$$\theta_5=\varphi-\mathrm{atan2}\left(\frac{x}{\rho},\ \pm\sqrt{1-\frac{x^2}{\rho^2}}\right)$$

We have

$$\theta_5=\mathrm{atan2}(\cos\theta_1\cos\alpha,\sin\theta_1)-\mathrm{atan2}\left(x,\pm\sqrt{(\cos\theta_1\cos\alpha)^2+\sin^2\theta_1-x^2}\right)\tag{5}$$

Secondly, define

$$c=\cos\theta_1\sin\alpha, d=\sin\theta_1\sin\theta_5+\cos\theta_1\cos\theta_5\cos\alpha$$

Eq. (3) then becomes

$$y=c\cos\psi-d\sin\psi$$

Let $d=\lambda\cos\tau$, $c=\lambda\sin\tau$, where $\lambda=\sqrt{d^2+c^2}$, $\tau=\mathrm{atan2}(c,d)$, so we have

$$\sin(\tau-\psi)=\frac{y}{\gamma},\cos(\tau-\psi)=\pm\sqrt{1-\frac{y^2}{\gamma^2}}$$

At a further step,

$$\tau-\psi=\mathrm{atan2}\left(\frac{y}{\gamma},\pm\sqrt{1-\frac{y^2}{\gamma^2}}\right)$$

Then, we have

$$\psi=\mathrm{atan2}(c,d)-\mathrm{atan2}\left(y,\ \pm\sqrt{d^2+c^2-y^2}\right)\tag{6}$$

2.2. Calculation of longitude

Longitude of observation point is related to time. Time T herein is used as GMT T_g for calculation. The calculation of longitude can be divided into three steps: (1) observation

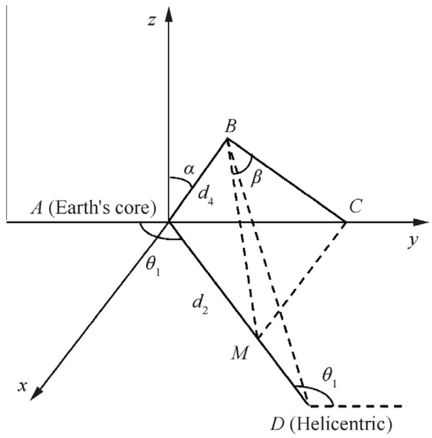

Fig. 2 Angle relation diagram of line and plane for longitude calculation.

position is firstly assumed as a Greenwich area. Hypothetical longitude θ_{5g} can be calculated by formula of angle between line and plane in space as shown in Fig. 2; (2) according to a known solar vector (direction of sunlight at this moment), the actual θ_5 can be calculated by Eq. (5); (3) considering difference between hypothetical θ_{5g} and actual θ_5, time difference between Greenwich area and actual observation area is applied to achieve a true longitude.

The detailed calculation of hypothetical θ_{5g} is shown as follows.

As shown in Fig. 2, angle β between line BC and plane ABD can be worked out by

$$\sin\beta = \frac{\sin\theta_1}{\sqrt{\cos^2\alpha + \sin^2\alpha\sin^2\theta_1}} \qquad \beta \in [0^\circ, 90^\circ]$$

Adding the solar hour angle $t = 15 \times (T_g - 12)$ and dividing it into different situations, we have

$$\begin{cases} \theta_{5g} = (180^\circ + \beta) + t & \theta_1 \in [0^\circ, 90^\circ] \\ \theta_{5g} = (360^\circ - \beta) + t & \theta_1 \in (90^\circ, 180^\circ] \\ \theta_{5g} = (0^\circ - \beta) + t & \theta_1 \in (180^\circ, 270^\circ] \\ \theta_{5g} = (180^\circ + \beta) + t & \theta_1 \in (270^\circ, 360^\circ] \end{cases} \tag{7}$$

where t is the solar hour angle in GMT.

In this case, the longitude can be expressed as

$$\gamma = \theta_5 - \theta_{5g} \tag{8}$$

3. Theoretical verification and comparative analysis

A comparative analysis is made in this section, in which, Michalsky's and Wang's formulas are chosen as known solar vectors' basis in the following examples. In detail, latitude calculation formulas were derived from Michalsky's sun azimuth and elevation angle formulas[23] and Wang and Liu's solar declination formulas.[24] The longitude calculation is calculated based on the time difference by the radio time. This contrastive algorithm is used as a standard reference and its errors of longitude and latitude are set to be zero.

The solar vector calculation parameters by the astronomy formulas are found from public resources. The solar vector calculation is generally based on the sun azimuth A and elevation angle H, as

$$\sin H = \sin\psi\sin\delta + \cos\psi\cos\delta\cos\omega \tag{9}$$

$$\sin A = \frac{\cos\delta\sin\omega}{\cos H} \tag{10}$$

where ω is the solar hour angle in local time, it equals 0° at noon, is negative in the morning and positive in the afternoon, and δ is the solar declination whose value is put forward by Wang and Liu[24] as

$$\begin{aligned} \delta = {} & 0.3723 + 23.2567\ \sin\theta + 0.1149\sin 2\theta - 0.1712\sin 3\theta \\ & - 0.785\cos\theta + 0.3656\cos 2\theta + 0.0201\cos 3\theta \end{aligned} \tag{11}$$

where

$$\begin{cases} \theta = 2\pi t_a/365.2422 \\ t_a = N_a - N_0 \end{cases}$$

where N_a is the day of year, the cumulative number of days from January 1 to the date in this year;

$$\begin{aligned} N_0 = {} & 79.6764 + 0.2422(\text{year} - 1985) \\ & - \text{INT}[0.25(\text{year} - 1985)] \end{aligned} \tag{12}$$

where INT represents the integer-valued function. Besides that, sun azimuth A and elevation angle H also need to be made an indirect projection in the coordinate system, and the true solar vector expression can be given as

$$\boldsymbol{S}_t = [\cos H\sin A, \cos H\cos A, \sin H]^T \tag{13}$$

This expression is selected to calculate known solar vectors. The time is as GMT and the revolution angle θ_1 can be obtained from relational expression with the number of days N away from the date of perihelion.

Considering symmetries of the earth as a globe and multiple roots of trigonometric functions, there exist multiple solutions in positioning calculation with a known solar vector. There would have more if the earth is assumed to be transparent to receive sunlight no matter in nighttime or daytime. Thus, an elimination of multiple solutions is needed to reach the expected one. This elimination has four phases. Firstly, latitude and longitude of a solution need to be transformed into standard value interval [−180°, 180°]. Periodical solutions are removed to avoid repeated roots due to trigonometric functions. Secondly, latitude coordinates out of value interval [−90°, 90°] have to be eliminated as definition in geography. It is noticed that those eliminated latitude coordinates cannot be further transformed into [−90°, 90°] since sine and cosine function are used simultaneously. Thirdly, a rough judgment of local time, such as day or night, morning or afternoon, can help to locate an actual local time calculated from two possible longitude coordinates. It is easy to do this through observing sun direction in local coordinate system (see if the sun is closer to east than to west or not). Finally, the left longitude coordinate related to the right local time is achieved. This longitude coordinate and the corresponding latitude coordinate are grouped to be the only solution depending on the given solar vector.

Note: the same sign in three examples is unrelated and data is independent; eliminated unit is (°).

3.1. Example 1

Comparative data: Tianjin,

$\gamma = 117^\circ$ E, $\psi = 39^\circ$ N

Known data:
GMT: April 5, 2014, 7:12 a.m.
Revolution angle: $\theta_1 = 105^\circ$, calculated by computer.
Solar vector is obtained from the astronomy Eqs. (9)–(12) and calculated by computer with local time, longitude and latitude, and the results are as follows:

$$\boldsymbol{S}_{t1} = [0.703361, 0.362755, 0.611304]^{\mathrm{T}}$$

3.1.1. Calculation of latitude and longitude

At first, substituting the known data into Eqs. (5) and (6) gives

$$\begin{cases} \theta_{51} = 58.810692^\circ \\ \psi_{11} = -39.001987^\circ \\ \psi_{12} = -157.649437^\circ \end{cases}, \begin{cases} \theta_{52} = 328.813748^\circ \\ \psi_{21} = -202.350563^\circ \\ \psi_{22} = 39.001987^\circ \end{cases}$$

In the above expression, the subscript is only to distinguish the difference between the calculated data. In θ_{51} and θ_{52}, 51 stands for the first group and 52 stands for the second group. In ψ_{11} and ψ_{12}, 11 stands for the first data in the first group data and 12 stands for the second data in the first group data. The following definitions are the same. According to the latitude' interval [−90°, 90°], we can eliminate unreasonable data and get

$$\begin{cases} \theta_{51} = 58.810692^\circ \\ \psi_{11} = -39.001987^\circ \end{cases} \tag{14a}$$

$$\begin{cases} \theta_{52} = 328.813748^\circ \\ \psi_{22} = 39.001987^\circ \end{cases} \tag{14b}$$

Secondly, taking location's GMT into Eq. (7),

$$\theta_{5g} = 211.820^\circ$$

Using Eq. (8),

$$\begin{cases} \gamma_1 = \theta_{51} - \theta_{5g} = -153.009308^\circ \text{ W} \\ \gamma_2 = \theta_{52} - \theta_{5g} = 116.993748^\circ \text{ E} \end{cases} \tag{15}$$

According to longitude solutions in Eq. (15) and the known GMT, the local time can be worked out and the answer is $T_1 = 21{:}\,00$ and $T_2 = 15{:}\,00$. Since sunlight cannot be seen at night 21:00 (it is not the calculating time we set in this case, but it may be a calculating time in other cases), some multiple solutions can be eliminated. Then the unique longitude coordinate can be obtained as $\gamma_2 = 116.993748^\circ$ E. Based on this achieved longitude coordinate and Eq. (14), the unique latitude coordinate is located as $\psi_{22} = 39.001987^\circ$ N. The latitude and longitude coordinate are then shown as

$$\begin{cases} \psi_{22} = 39.001987^\circ \text{ N} \\ \gamma_2 = 116.993748^\circ \text{ E} \end{cases} \tag{16}$$

3.1.2. Error calculation

Making a difference between calculated latitude and longitude solution Eq. (16) and actual comparative data, we have

$$\begin{cases} \Delta\psi = \psi_{22} - \psi = 0.001987^\circ \\ \Delta\gamma = \gamma_2 - \gamma = -0.006252^\circ \end{cases} \tag{17}$$

3.2. Example 2

Comparative data: London,

$\gamma = 0^\circ$W,$\psi = 51.05^\circ$N

Known data:
GMT: March 21, 2014, 9: 00 a.m.
Revolution angle: $\theta_1 = 90^\circ$
Solar vector:

$$\boldsymbol{S}_{t2} = [-0.707107, 0.549526, 0.444995]^{\mathrm{T}}$$

which is obtained from the astronomy formulas and calculated by computer.

3.2.1. Calculation of latitude and longitude

At first, the known data are applied to Eqs. (5) and (6), which gives

$$\begin{cases} \theta_{51} = 135.000018^\circ \\ \psi_{11} = -51.000128^\circ \\ \psi_{12} = -128.999872^\circ \end{cases}, \begin{cases} \theta_{52} = 224.999983^\circ \\ \psi_{21} = 128.999872^\circ \\ \psi_{22} = 51.050112^\circ \end{cases}$$

According to the latitude' interval [−90°, 90°], we can eliminate unreasonable data and get

$$\begin{cases} \theta_{51} = 135.000018^\circ \\ \psi_{11} = -51.000128^\circ \end{cases} \tag{18a}$$

$$\begin{cases} \theta_{52} = 224.999982^\circ \\ \psi_{22} = 51.050112^\circ \end{cases} \tag{18b}$$

Secondly, GMT is taken into Eq. (7),

$$\theta_{5g} = 225.000^\circ$$

Using Eq. (8) further, we have

$$\begin{cases} \gamma_1 = \theta_{51} - \theta_{5g} = -90.000018^\circ \text{ W} \\ \gamma_2 = \theta_{52} - \theta_{5g} = -0.000018^\circ \text{ W} \end{cases} \tag{19}$$

According to longitude solutions in Eq. (19) and the GMT, the local time can be calculated as $T_1 = 3{:}\,00$ and $T_2 = 9{:}\,00$. Since a day time but a night time is selected, some multiple solutions can be eliminated. Then the unique longitude solution can be obtained as $\gamma_2 = -0.000018^\circ$ W. Based on longitude result and Eq. (18), the left latitude solution is, $\psi_{22} = 51.050112^\circ$ N. The latitude and longitude coordinate is then

$$\begin{cases} \psi_{22} = 51.050112^\circ \text{ N} \\ \gamma_2 = -0.000018^\circ \text{ W} \end{cases} \tag{20}$$

3.2.2. Error calculation

Making a difference between calculated latitude and longitude data Eq. (20) and actual comparative data, we can get

$$\begin{cases} \Delta\psi = \psi_{22} - \psi = 0.000112^\circ \\ \Delta\gamma = \gamma_2 - \gamma = -0.000018^\circ \end{cases} \tag{21}$$

3.3. Example 3

Comparative data: New York,

$\gamma = -74^\circ$ W, $\psi = 40.433^\circ$ N

Known data:
GMT: December 22, 2013, 16: 56 p.m.
Revolution angle: $\theta_1 = 0^\circ$
Solar vector:

$$\boldsymbol{S}_{t3} = [0, 0.897744, 0.440518]^{\mathrm{T}}$$

which is obtained from the astronomy formulas and calculated by computer.

3.3.1. Calculation of latitude and longitude

At first, known data are applied to Eqs. (5) and (6) gives

$$\begin{cases} \theta_{51} = 0^\circ \\ \psi_{11} = -40.429793^\circ \\ \psi_{12} = -92.703607^\circ \end{cases}, \begin{cases} \theta_{52} = 180^\circ \\ \psi_{21} = 92.703607^\circ \\ \psi_{22} = 40.429793^\circ \end{cases}$$

According to the latitude' interval [−90°, 90°], we can eliminate unreasonable data and get

$$\begin{cases} \theta_{51} = 0^\circ \\ \psi_{11} = -40.429793^\circ \end{cases} \tag{22a}$$

$$\begin{cases} \theta_{52} = 180^\circ \\ \psi_{22} = 40.429793^\circ \end{cases} \tag{22b}$$

Secondly, GMT is used in Eq. (7),

$$\theta_{5g} = 253.999995^\circ$$

Using Eq. (8), we have

$$\begin{cases} \gamma_1 = \theta_{51} - \theta_{5g} = 106.000005^\circ \text{ E} \\ \gamma_2 = \theta_{52} - \theta_{5g} = -73.999995^\circ \text{ W} \end{cases} \tag{23}$$

According to the longitude in Eq. (23) and the GMT, the local time can be calculated, $T_1 = 24{:}\ 00$ and $T_2 = 12{:}\ 00$. Since a day time is chosen, some multiple solutions can be eliminated. Then the unique longitude coordinate can be obtained as $\gamma_2 = -73.999995^\circ$ W. Based on longitude result and Eq. (22), solutions of latitude can be eliminated and the unique latitude coordinate is achieved, $\psi_{22} = 40.429793^\circ$ N. The latitude and longitude coordinates are shown as follows:

$$\begin{cases} \psi_{22} = 40.429793^\circ \text{ N} \\ \gamma_2 = -73.999995^\circ \text{ W} \end{cases} \tag{24}$$

3.3.2. Error calculation

Making a difference between calculated latitude and longitude data Eq. (24) and actual comparative data, we can get

$$\begin{cases} \Delta\psi = \psi_{22} - \psi = 0.003540^\circ \\ \Delta\gamma = \gamma_2 - \gamma = -0.000005^\circ \end{cases} \tag{25}$$

4. A test via a designed iOS application

Smart mobile device leads the trend of modern life. It becomes an important tool platform to supply many services. In order to test the feasibility of the proposed positioning theory and build a base of special instrument, a smart phone positioning application is developed and introduced in this section. The development platform is based on Apple's iOS system.

4.1. Realization principle

Current smart phones, like iPhone, are equipped with many useful sensors. For example, gyroscope (Gyro) and magnetic sensor can be used to provide phone's attitude in the space, which can be used by the proposed algorithm as an input data for positioning. Also, a mini global position system (GPS) receiver has been embedded into a smart phone, by which, the WGS84 coordinates of device's position can be shown on the screen in real time. Hence, GPS data should be a good comparative object to the output of the developed application based on the proposed algorithm. Besides, a phone camera can help to find the sun. Thus, all basic parameters including solar altitude, geomagnetic azimuth and GMT can be acquired by an iPhone for the proposed method.

Values of solar zenith angle, azimuth angle and geomagnetic azimuth angle are achieved and calculated firstly. According to the definition of iPhone's space coordinates, the attitude information of the phone in the space can be obtained by those angles. The axis of acceleration sensors and Gyro are shown in Figs. 3 and 4 respectively, which can be found in Alan's book.[25]

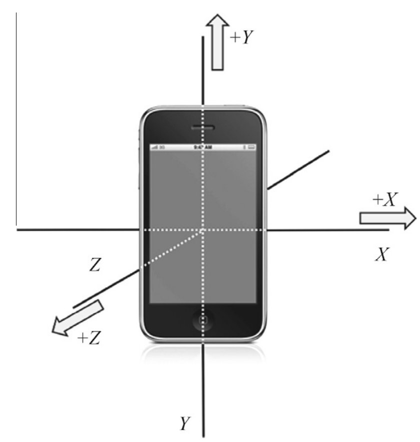

Fig. 3 Axis of acceleration sensors.

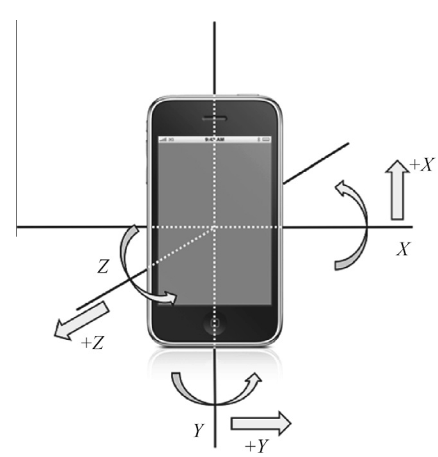

Fig. 4 Three-axis digital Gyro setting in the Phone.

A phone's camera actually consists of optic sensors which can be used to determine the relative direction of the sun about the phone's screen. Based on optic principle, F is the intersection of camera imaging when the sun's rays are perpendicular to the camera (see Fig. 5). When the sun is located in field of view of phone's back camera, elevation angle can be determined. At the same time, the geomagnetic direction angle of sun can be calculated with space vector algebra. Since GMT of a smart phone is now accurately supplied by time service center, positioning time is recorded by the developed application.

Through the above steps, three input parameters are obtaind for the proposed algorithm. These manipulations and the proposed algorithm are integrated to developed phone's application. So, latitude and longitude coordinates can be obtained from this celestial body positioning method. As it can be seen in Fig. 5, a red dot is designed to locate position of sun in the field of view. As shown in Fig. 6, by opening the application switch, lay the sun into red dot on the phone's screen and press LOCK button, positioning calculation is completed and shown on the screen. All the units of the angles in the figures are (°).

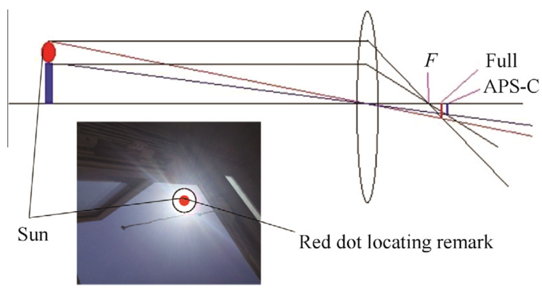

Fig. 5 Camera positioning principle.

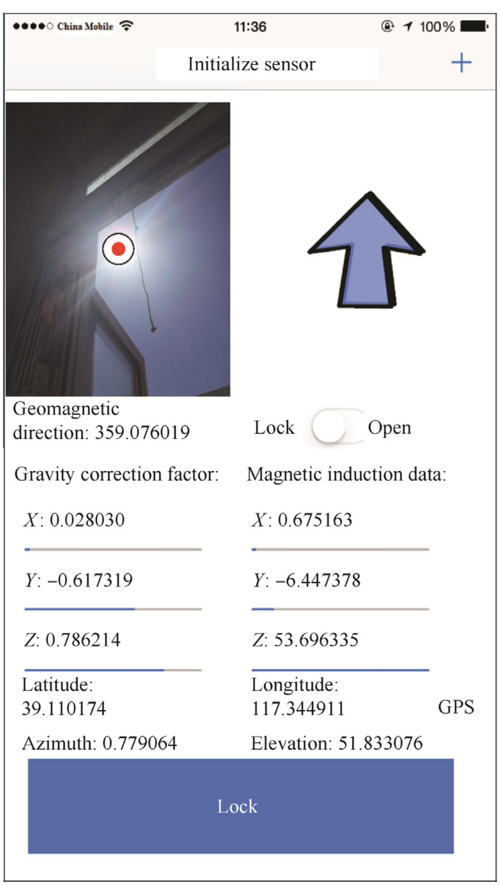

Fig. 6 Positioning application interface.

4.2. Data processing and deviation

In order to obtain more accurate spatial attitude data and correcte accelerometer bias caused by hand jitter in measurement, a method for the mutual correction of the gyroscope and accelerometer is used to obtain accurate gravity direction data, which makes the calculation more accurate in developed application. As shown in Fig. 7, GPS data of the same position is also displayed in the result screen as a comparative object.

5. Discussions and conclusions

According to positioning methodology in Section 2, theoretical comparison in Section 3, and practical test in Section 4, the following discussions and conclusions can be drawn:

First of all, the calculation errors of latitude and longitude are about a few thousandths in angle. Considering the ignored tiny value d_6/d_2 in this model, error of astronomy formula and the effective digital selection in the process of calculation, error results are considered to be reasonable. That is to say, the presented expressions of latitude and longitude by theory of mechanisms are correct and applicable. As it can be seen, performance of the proposed algorithm is essentially equivalent to the contrastive algorithm. Besides, the proposed method breaks up the fixed thinking mode of calculating celestial coordinates with astronomical trigonometry and simplifies the complexity of parameters calculation. Especially, the method only makes use of GMT and solar vector of observation area.

To its accuracy, by calculating arc-length of the earth circle, $L = \Delta \times \pi \times r/180$ (Δ is result error of latitude and longitude; r is radius of the earth), theoretical distance error of the proposed positioning method can be calculated with error results Eqs. (17), (21) and (25). It shows that the maximum distance error is about 0.7 km, approximately equals 0.4 n mile. At this level, it is equivalent to former articles and can be acceptable by PBN requirements. Besides, the precision of the proposed method can be further reduced by high-precision observation instruments or combined with an inertia navigation system. Hence, the proposed method meets the basic requirements of navigation application in civil aviation.

Elevation angle: 51.833076
Deviation angle with geomagnetic south pole: 0.779064
Calculation latitude: 38.383319
Calculation longitude: 119.487015
GPS latitude: 39.110174
GPS longitude: 117.344911

Fig. 7 Interface of positioning results and comparative GPS data.

To test its practicality and robustness, a phone application is developed and utilized. The positioning application that originated from the proposed algorithm is tested by hundred times. According to one of these tests, as it shown in Fig. 7, position coordinates are obtained by the proposed method and compared to GPS data in the same position at the same time. It shows a certain deviation in calculated coordinates to GPS data, but within a certain range. Hence, the proposed positioning method is applicable and stable. It needs to be gradually modified though. There are reasons for deviation in current application, such as: measurement error of phone's sensors, our simple but rough way to locate sun's direction, earth's geomagnetic variety change, the atmospheric refraction of sun's rays (deviation of morning and evening is more obvious), deviation of WGS84 coordinates and Chinese GCJ02 coordinates (normally about 300–500 m), etc. To improve the accuracy of this application in an effective way, a bias data in every usage should be recorded and uploaded to a cloud database and get revised for the next use if it can be used by a large number of customers in different locations. Or, a special instrument that originates from proposed method is looked forwards in the future.

Finally, the proposed algorithm can be applied to the calculation of local time, latitude and longitude. In this way, positioning time can be saved greatly and the long-period's problem in sun sight running fixing process can be solved. Even during the daytime, a continuous positioning can be realized. It is believed that single celestial sun positioning will have a better application prospect. Its methodology of using theory of mechanisms can inspire development of new sun positioning device in both application and hardware.

Acknowledgements

This study was co-supported by the National Natural Science Foundation of China, Civil Aviation Administration of China (Nos. U1233106, U1533103, 11502284), the Fundamental Research Funds for the Central Universities (No. ZXH2012H007) and Tianjin Natural Science Foundation (No. 15JCQNJC42600).

References

1. Wu HX, Yu WB, Fang JC. Simulation of SINS/CNS integrated navigation system used on high altitude and long-flight-time unpiloted aircraft. *Acta Aeronaut et Astronaut Sin* 2006;**27**(2):299–304 [Chinese].
2. Wang XL, Ma S. A celestial analytic positioning method by stellar horizon atmospheric refraction. *Chin J Aeronaut* 2009;**22**(3):293–300.
3. Miao LJ, Shi J. Model-based robust estimation and fault detection for MEMS-INS/GPS integrated navigation systems. *Chin J Aeronaut* 2014;**27**(4):947–54.
4. Zheng XQ, Deng XG, Yang XX. Research on the error model of airborne celestial/inertial integrated navigation system. *Proceedings of the international conference on photonics and optical engineering*; 2014 Oct 13–15; Xi'an, China. Bellingham (WA): SPIE; 2015. p. 1–9.
5. Huang J, Gao HR, Dai JX. Performance-based navigation (PBN) impact the instrument flight procedure implementation in the civil aviation of China. *J Wuhan Univer Technol (Transportation Science & Engineering)* 2011;**35**(4):787–92 [Chinese].
6. Xia JB, Chen YL, Wang ZZ. *Practical Astronomy*. Wuhan: Wuhan University Press; 2007. p. 109–228 [Chinese].
7. Wang SX, Lian CX, Liu X. Single celestial positioning method study. *Proceedings of 2002 symposium on navigation practical new technology*; 2002; Dalian, China. Beijing: China Academic Journal Electronic Publishing House; 2002. p. 150–2 [Chinese].
8. Tang ZP, Yang XD. A method of astronomy fix by simple celestial body. *Navigation Technol* 2002;**3**:20–1 [Chinese].
9. Jiang J, Yin Y. Study on the simulation of celestial fixing in navigation simulator. *J Syst Simul* 2008;**20**(Suppl):63–6 [Chinese].
10. Jia HH, Wu JH, Zhao X, Ma Z. Computer-aided single celestial-body positioning and precision analysis. *J Tianjin Univer Technol* 2009;**25**(6):31–4 [Chinese].
11. Yuan X, Zhang X. Research and design of non-GPS positioning system based on solar orientation algorithm. *China Water Transport* 2011;**11**(8):68–70 [Chinese].
12. Wang GH, Guo ZD, Zhu H, Mo J. Capability analysis of polarized light celestial positioning. *Acta Photonica Sinica* 2012;**41**(1):11–4 [Chinese].
13. Peng CL, Wang GL, Chen XZ, Luo CW, Wen M. Design of automatic calculation system for navigation astronomical orientation. *J Hunan Univer Sci Technol* 2013;**28**(4):82–5 [Chinese].
14. Zhao X, Xing EH, Wu JH, Jia HH. Anti-midlines with inner method enhance positioning accuracy in navigation by three celestial-bodies. *Proceedings of 2010 international conference on computer design and applications (ICCDA 2010)*; 2010 Jun 25–27; Qinhuangdao, China. Piscataway (NJ): IEEE Press; 2010. p. 294–7.
15. Bernhardt MG, Becker W, Prinz T, Breithuth FM, Walter U. Autonomous spacecraft navigation based on pulsar timing information. *Proceedings of the 2nd international conference on space technology (ICST)*; 2011 Sep 15–17; Athens, Greece. Piscataway (NJ): IEEE Press; 2011. p. 1–4.
16. Wang J. An orientation method and analysis of optical radiation sources based on polyhedron and parallel incident light. *Sci China Tech Sci* 2013;**43**(2):161–8.
17. Dalvand MM, Shirinzadeh B, Nahavandi S. Inverse kinematics analysis of 6-RRCRR parallel manipulators. *Proceedings of 2013 IEEE/ASME international conference on advanced intelligent mechatronics (AIM)*; 2013 July 9–12; Wollongong, Australia. Piscataway (NJ): IEEE Press; 2013. p. 644–8.
18. Atique MMU, Ahad MAR. Inverse kinematics solution for a 3DOF robotic structure using Denavit-Hartenberg convention. *Proceedings of 2014 international conference on informatics, electronics & vision 2014 (ICIEV)*; 2014 May 23–24; Dhaka, Bangladesh. Piscataway (NJ): IEEE Press; 2014. p. 1–5.
19. Gao SS, Gao Y, Zhong YM, Wei WH. Random weighting estimation method for dynamic navigation positioning. *Chin J Aeronaut* 2011;**24**(3):318–23.
20. Chen IM, Yang GL, Chee TT, Song HY. Local POE model for robot kinematic calibration. *Mech Mach Theory* 2001;**36**(11):1215–39.
21. Zhou FF, Fan XP, Ye Z. 3-D virtual robotic model generated by D-H parameters. *J Syst Simul* 2006;**18**(4):947–50.
22. Zhang QZ, Zhou YL. *Simple tutorial of robotics*. Xi'an: Xi'an University of Electronic Science and Technology Press; 2013. p. 39–44 [Chinese].
23. Michalsky JJ. The astronomical almanac's algorithm for approximate solar position (1950–2050). *Sol Energy* 1988;**40**(3):227–35.
24. Wang BZ, Liu GS. Improvement in the astronomical parameters computation for solar radiation observation. *Acta Energi Sin* 1991;**12**(1):27–32 [Chinese].
25. Allan A. *Basic sensors in IOS: Programming the accelerometer, gyroscope, and more*. Sebastopol (CA): O'Reilly Media Inc.; 2011. p. 46–79.

Zhang Wei received the Ph.D. degree from Tianjin University. He is an associate professor at Civil Aviation University of China. His research area includes mechanism and robot.

Xu Xiaofeng is a postgraduate at Aeronautical Automation College, Civil Aviation University of China. His research area is mechanism and robot.